Homework 7 Solution

Problem 7.2   Cost of buying is B and renting is R. Consider a strategy of renting till time $t-1$ and buying if t-th ski trip happened. The worst case data for this strategy is if there are exactly t ski trips, in which case the cost of this algorithm is $(t-1)R + B$. For the same data, the optimal algorithm pays the cost of $\min\{Rt, B\}$. The competitive ratio is then $CR(t) = \frac{(t-1)R+B}{\min\{Rt,B\}}$, which is minimized at either $t = \lfloor B/R \rfloor$ or $t = \lceil B/R \rceil$. For simplicity, let's assume that B/R is an integer. Then, this algorithm achieves the competitive ratio of

$$CR = \frac{2B - R}{B} = 2 - \frac{R}{B} \leq 2 .$$

Problem 7.2

(a) When $J \leq M$, all machines have at most one job assigned, and hence the maximum completion time is $\max_j t_j$. There is no assignment that can complete all jobs in shorter time.

(b) Again, in this case the maximum completion time is $\max_j t_j$. There is no assignment that can complete all jobs in shorter time.

(c) By definition of the greedy algorithm, when job j is assigned to machine i, machine i must be the one with minimum load at that time, i.e.

$$GA(L) - t_j \leq \frac{1}{M} \sum_{k=1}^{j-1} t_k \leq \frac{1}{M} \sum_{i=1}^{M} T_i$$

(d) Since there is at least one machine with two jobs, each at least t_{M+1}, even the optimal assignment has the maximum completion time $OPT(L) \geq 2t_{M+1}$.

(e) In the best case, when all machines are equally occupied in term of completion time, then the maximum completion time is equal to the average completion time. In any other case, it is only larger even for optimal assignment.

(f) Due to (a) and (b), we can assume that $J > M$ and the machine with maximum completion time under greedy algorithm has at least two jobs. Otherwise, we know that the greedy algorithm is optimal. It follows that

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\begin{aligned}
GA(L) \;\; &\leq \;\; t_j + \frac{1}{M} \sum_{i=1}^{M} T_i \\
&\leq \;\; \frac{1}{2} OPT(L) + OPT(L) = \frac{3}{2} OPT(L) .
\end{aligned}
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