Achieving budget-optimality with adaptive schemes in crowdsourcing

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joint work with Ashish Khetan
(David Karger, Devavrat Shah, Jungseul Ok, Jinwoo Shin, Yung Yi)
Adaptive task assignment for crowdsourced classification

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Crowdsourcing systems
Marketplace to get labels for training data

airplanes?

Quality of the labels can be very low

70/100 30/100
Add redundancy to cope with noise
Add redundancy to cope with noise
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Tradeoff: redundancy vs. accuracy

- special to crowdsourcing system:
  - we pay for each response
  - we design the graph
  - workers arrive online fashion
Notations

- $m$ questions
- unknown true answers $t \in \{-1, +1\}^m$
- task-assignment graph $G([m], [n], E)$
  
  $(i, j) \in E$ indicates that question $i$ is asked to the $j$-th arriving worker
- $n$ workers arrive online fashion and submit responses
- response matrix $A \in \{-1, 0, 1\}^{m \times n}$

\[
A_{ij} = \begin{cases} 
0 & \text{if not assigned} \\
+1 & \text{if worker answers "+1"} \\
-1 & \text{if worker answers "−1"}
\end{cases}
\]

- Dawid-Skene model from 1979:
  - Each worker is parametrized by a scalar value $p_j \in [0, 1]$
  - For each assigned question, answers correctly with probability $p_j$

\[
A_{ij} = \begin{cases} 
t_i & \text{with probability } p_j \\
-t_i & \text{with probability } 1 - p_j
\end{cases}
\]

- Criticism: all tasks are assumed to be equally difficult
Design:

1. Task assignment graph $E$
2. Inference algorithm $\hat{t}(A) \in \{-1, +1\}^m$

How does error rate trade-off with cost/redundancy?

$$
\mu \triangleq \frac{1}{n} \sum_{j=1}^{n} (2p_j - 1)
$$

$$
\sigma^2 \triangleq \frac{1}{n} \sum_{j=1}^{n} (2p_j - 1)^2
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![Task assignment graph](image)

### Majority Voting

$$
\sim e^{-c\mu^2 \ell}
$$

### Oracle Estimator

$$
\sim e^{-c'\sigma^2 \ell}
$$
Design:

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<table>
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<tr>
<th></th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p_5$</th>
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<td>$t_5$</td>
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<table>
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<tr>
<th>Cost (Number of assignments per task $\ell$)</th>
<th>Probability of error</th>
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<tr>
<td>5</td>
<td>1</td>
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<tr>
<td>10</td>
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<td>15</td>
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<td>0.0001</td>
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<td>30</td>
<td>1e-05</td>
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</table>

- Majority Voting $\sim e^{-c\mu^2\ell}$
- Iterative Algorithm $\sim e^{-c^\prime\sigma^2\ell}$
- Oracle Estimator $\sim e^{-c^\prime \sigma^2 \ell}$
What is known for DS model

\[ \sigma^2 \triangleq \frac{1}{n} \sum_{j=1}^{n} (2p_j - 1)^2 \]

**Achievability:** random graph \( E \) and iterative inference achieves

\[ P_{err} \leq e^{-c\sigma^2\ell} \]

**Fundamental limit:** the best task-assignment and inference is limited by

\[ \min_{E,\hat{t}} \max_{t \in \{\pm 1\}^m, p \in F_{\sigma^2}} P_{err} \geq e^{-c'\sigma^2\ell} \]
What is known for DS model

quality of the crowd \( \sigma^2 \triangleq \frac{1}{n} \sum_{j=1}^{n} (2p_j - 1)^2 \)

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**Fundamental limit:** the best adaptive is limited by

\[
\min_{\text{all adaptive schemes}, \hat{t}} \max_{t \in \{\pm 1\}^m, p \in F_{\sigma^2}} P_{\text{err}} \geq e^{-c''\sigma^2 \ell}
\]
Adaptive schemes are common in practice

- in practice, adaptive schemes improve significantly
- in theory, the gain is minimal
Adaptive schemes are common in practice

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[Images of different landscapes and environments]
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When does adaptivity help?
Generalized DS model [Zhou et al. ’15]

Definition

- Each task is parametrized by a scalar value $q_i \in [0, 1]$
- When a task is presented to a worker, it is perceived as a positive task with probability $q_i$

$$A_{ij} = \begin{cases} +1 & \text{with probability } q_ip_j + (1 - q_i)(1 - p_j) \\ -1 & \text{with probability } q_i(1 - p_j) + (1 - q_i)p_j \end{cases}$$

- ground truth is $t_i = \mathbb{I}(q_i > 0.5) - \mathbb{I}(q_i < 0.5)$
- difficulty level of a task $i$ measured by $(2q_i - 1)^2$
Generalized DS model [Zhou et al. ’15]

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Generalized DS model with adaptive scheme

Collective task difficulty:

$$\rho^2 = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{(2q_i - 1)^2}$$

![Graph showing the relationship between redundancy and error probability for non-adaptive and adaptive inference methods.]

- Non-adaptive $E$
  - Majority voting
- Non-adaptive $E$
  - Iterative inference
  - $\approx e^{-c(2q_{\text{min}} - 1)^2 \sigma^2 \ell}$
- Adaptive $E$
  - Iterative inference
  - $\approx e^{-c' \rho^2 \sigma^2 \ell}$
Minimax rate for adaptive scenario

- Task difficulty: 
  \[ \rho^2 = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{(2q_i - 1)^2} \]

Fundamental limit: the best adaptive scheme and inference is limited by

\[ \min_{\text{adaptive } E, \hat{t}} \max_{q \in G_{\rho^2}, p \in F_{\sigma^2}} P_{\text{err}} \geq e^{-C' \rho^2 \sigma^2 \ell} \]

Achievability: efficient adaptive scheme and inference achieves

\[ P_{\text{err}} \leq e^{-C \rho^2 \sigma^2 \ell} \]

Fundamental limit: the best non-adaptive scheme is limited by

\[ \min_{E, \hat{t}} \max_{q \in G_{\rho^2}, p \in F_{\sigma^2}} P_{\text{err}} \geq e^{-C'' (2q_{\text{min}} - 1)^2 \sigma^2 \ell} \]
Message-passing algorithm

- Two sets of messages:
  - Task messages \{\text{T}_i \rightarrow j\}, and worker messages \{\text{W}_j \rightarrow i\}
- Initialize worker messages as random Gaussian: \text{W}_j \rightarrow i \sim \mathcal{N}(1, 1)
- Iteratively update messages

Task-likelihood update

\[
\text{T}_{i \rightarrow j} = \sum_{j' \neq j} \text{W}_{j' \rightarrow i} \text{A}_{ij'}
\]

Worker-reliability update

\[
\text{W}_{j \rightarrow i} = \sum_{i' \neq i} \text{T}_{i' \rightarrow j} \text{A}_{i'j}
\]

A task is likely to be ‘+’ if reliable workers agree that it is ‘+’

A worker is reliable if the worker agreed with our belief on other tasks
1. DS model + non-adaptive scheme
1. DS model + non-adaptive scheme

\[ P(T_{i\rightarrow j}|q_i = 0) \]

\[ P(T_{i\rightarrow j}|q_i = 1) \]

\[ T_{i\rightarrow j} = \sum A_{ik} W_{k\rightarrow i} \]
1. DS model + non-adaptive scheme

\[ P(T_{i \rightarrow j} | q_i = 0) \]

\[ P(T_{i \rightarrow j} | q_i = 1) \]
1. DS model + non-adaptive scheme

\[ \mathbb{P}(T_{i\rightarrow j}|q_i = 0) \]

\[ \mathbb{P}(T_{i\rightarrow j}|q_i = 1) \]

\[ -1 \quad 0 \quad +1 \]

\[ 2q_i - 1 \]
1. DS model + non-adaptive scheme

\[ \mathbb{P}(T_{i \rightarrow j} | q_i = 0) \]
\[ \hat{t}_i = -1 \quad \hat{t}_i = +1 \]

\[ \mathbb{P}(T_{i \rightarrow j} | q_i = 1) \]

\[ \frac{1}{\sqrt{\sigma^2 \ell}} \]
1. DS model + non-adaptive scheme

\[ \mathbb{P}(T_{i \rightarrow j} | q_i = 0) \]

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\[ P_{\text{err}} \leq e^{-C \sigma^2 \ell} \]
2. Generalized DS model + non-adaptive scheme
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\[ P(T_{i \rightarrow j} | q_i) \]
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2. Generalized DS model + non-adaptive scheme

\[ \hat{t}_i = -1 \quad \hat{t}_i = +1 \]

\[ P_{err} \leq e^{-C(2q_{\min} - 1)^2 \sigma^2 \ell} \]
3. Generalized DS model + adaptive scheme

- Repeat rounds $t \in \{1, 2, \ldots\}$
  - Assign tasks with random $(\ell_t, r)$-regular random graph
  - Run iterative algorithm
  - Classify high-confidence tasks with threshold $\mathcal{X}_t$
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  - Classify high-confidence tasks with threshold $\mathcal{X}_t$
3. Generalized DS model + adaptive scheme

\[ \hat{t}_i = -1 \quad \text{or} \quad \hat{t}_i = +1 \]

- Repeat rounds \( t \in \{1, 2, \ldots\} \)
  - Assign tasks with random \((\ell_t, r)\)-regular random graph
  - Run iterative algorithm
  - Classify high-confidence tasks with threshold \( \lambda_t \)
Where did this algorithm come from?

- Spectral method

\[
A = \begin{bmatrix}
\frac{1}{n} \cdot t_i \cdot (2p_j - 1) & \frac{1}{n} \cdot t \cdot (2p - 1)^T
\end{bmatrix}
\]

- Random Perturbation

\[
E[A_{ij}|t_i, p_j] = \frac{\ell}{n} \cdot t_i \cdot (2p_j - 1)
\]

\[
E[A|t, p] = \frac{\ell}{n} \cdot t \cdot (2p - 1)^T
\]

- Singular vector of a non-backtracking matrix (widely used in community detection [Mossel et al. '13, Krzkala et al. '13, Bordenave et al. '14, Saade et al. '15, etc.])

- Belief propagation for approximating [Peng et al. 12]

\[
P(t_i|A)
\]
Recap

Dawid-Skene model

- Each worker with quality $p_j$

$$\sigma^2 = \frac{1}{n} \sum_{j=1}^{n} (2p_j - 1)^2$$

$$P_{err} \sim e^{-C\sigma^2\ell}$$

Criticism: homogeneous tasks

No gain in adaptivity
Recap

Dawid-Skene model

- Each worker with quality $p_j$
  \[ \sigma^2 = \frac{1}{n} \sum_{j=1}^{n} (2p_j - 1)^2 \]
  \[ P_{\text{err}} \approx e^{-C \sigma^2 \ell} \]

- Criticism: homogeneous tasks
- No gain in adaptivity

generalized Dawid-Skene

- Each task with difficulty $q_i$
  \[ \rho^2 = \frac{1}{m} \sum_{i=1}^{m} \frac{1}{(2q_i - 1)^2} \]
  \[ P_{\text{err}} \approx e^{-C \rho^2 \sigma^2 \ell} \]

- non-adaptive limit:
  \[ P_{\text{err}} \approx e^{-C (2q_{\text{min}} - 1)^2 \sigma^2 \ell} \]
Ongoing/future work

What about Belief Propagation and Expectation Maximization?

![Graph showing probability of error vs. number of workers per task]

- Majority voting
- Non-adaptive
- Lower bound

\[ P_{\text{BP, err}} - P_{\text{LB, err}} \leq m - \gamma \text{ for some } \gamma > 0 \]
Ongoing/future work

What about Belief Propagation and Expectation Maximization?

[Ok, Oh, Shin, Yi ’16]

\[ P_{\text{err}}^{\text{BP}} - P_{\text{err}}^{\text{LB}} \leq m^{-\gamma} \text{ for some } \gamma > 0 \]
Related work

- Crowdsourcing in machine learning
  - [ShengProvostIpeirotis2008] - first modern application of DS model
  - [WhitehillWuBergsmaMovellanRuvolo2009] - NIPS, release datasets
  - [WelinderBransonPeronaBelongie2010] - NIPS, release datasets
  - [GhoshKaleMcAfee2011] - first analysis of DS model $P_{\text{err}} \leq \frac{C}{\sigma^2 \ell}$
  - [HoJabbariVaughan2013] - online arrival of workers, mixture of DS model

- Lots of recent papers...

- In this talk...
  - *Optimality of Belief Propagation for Crowdsourced Classification*, Ok, O. Shin, Yi, ICML 2016